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DIVERGING AND CONVERGING SCHEMES OF APPROXIMATIONS FOR GAUSSIAN BEAMS

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Abstract

EM Gaussian beams are the most celebrated and used kind of laser beams, and their description for various applications, such as light scattering and optical particle characterization, has therefore a long and venerable history. To this aim, the Davis scheme of approximations has been one of the most celebrated. An overlooked paper has nevertheless established a quite unexpected result, namely that this scheme is actually divergent. Our presentation will discuss the main features of this divergence and recall the existence of convergence schemes which have been deduced from the divergent Davis scheme.

1 The Davis scheme and its divergence.

The Davis scheme of approximations relies on the introduction of an x-polarized potential vector [1-2], from which we may deduce electric and magnetic field components in the Lorentz gauge. The Gaussian beam is characterized by a small parameter denoted "s" and called the beam confinement factor (or beam shape factor). The x-component of the potential vector satisfies the Helmholtz equation. The solution is searched under the form of an infinite series in terms of even powers of "s". Only the first-order, third-order and fifth-order approximations which respectively contain terms up to $O(s^2)$, $O(s^4)$ and $O(s^6)$ are explicitly known. None of the approximations in the scheme satisfies Maxwell's equations which are only satisfied in the infinite-order limit. Nevertheless, the first three terms of the series are sufficient to obtain, in practice, a good description of a Gaussian beam. The convergence of the whole series, considering the small value of the beam confinement factor (0 for a plane wave, typically 10^{-3} for a moderate focusing, and 1/6 for an extreme focused case when the wave-length is about equal to the beam waist radius of the beam), has been considered as guaranteed. In a very overlooked paper [4], it has surprisingly been demonstrated that the series is actually divergent.

The demonstration of the divergence is sophisticated and relies on the study of a partial differential equation for the x-component of the potential vector whose solution is researched under the form of infinite series defined by recurrence relations between expansion coefficients. The study of the successive terms of the series shows that the series, to begin with, converges to a satisfactory solution, before diverging. We shall return to such a behaviour later.

2 Convergent schemes.

From the divergent Davis scheme, two convergent schemes have been however developed and are known since a long time.

The first one is the localized approximation schemes whose convergence is ensured by the fact that they produce closed-form expressions, whose validity in the case of Gaussian beams has been established both for on-axis configurations [5] and for off-axis configurations [6]. These closed-form expressions provide the evaluation of the beam shape coefficients which encode the structure of the beam and from which we may evaluate all field components of the Gaussian beam.

The second one relies on the evaluation of the beam shape coefficients directly from the first-order, third-order and fifth-order Davis approximations. It is found, after a significant amount of calculation, that for each of these cases, the beam shape coefficients are the summation of terms which do not depend on the coordinates and on non-constant terms which do depend on the coordinates. These coordinate-dependent terms are artefacts produced by the fact that the Davis beam approximations do not satisfy Maxwell's equations. Once they are removed, we obtain beam descriptions which exactly satisfy Maxwell's equations. For instance, in the simple case when the beam waist centre of the beam is located at the origin of the coordinates, the beam shape coefficients simply read as $[1-(n-1)(n+2)s^2]$ for the first-order Davis beams, and similar simple although longer expressions for the third-order and for the fifth-order Davis beams.

We have then defined standard beams as the infinite generalization of the beams defined by the first Davis beams when the artefacts are removed.

In a first version, called standard beams, we obtain a satisfactory description of Gaussian beams, but the series obtained possessed a limited radius of convergence. An improved standard beam scheme has afterward been designed whose convergence was guaranteed [7].

3 Complementary discussion

The divergent series of the Davis is an example of asymptotic series which is reminiscent of asymptotic series encountered in quantum electrodynamics. Such series are non-convergent series which however provide a correct result if we limit ourselves to a few terms. A paradigmatic example is the evaluation of the electron g-factor which is a dimensionless magnetic moment. It may evaluated by a series reading as $g/2=1+C(1)\alpha+C(2)\alpha^2 +\ldots$

In this series, α is a small parameter (the fine structure constant) equal to 1/137.035…, from which we might have expected a fast convergence of the series. Such is not the case however, and the calculations of the successive coefficients, relying on an evaluation of an increasing number of integrals related to Feynman diagrams, become more and more complicated. For instance, the calculation of C(3) requires the evaluation of 72 integrals while C(4) requires the evaluation of 891 integrals [8]. The theoretical value, obtained by summing only such few terms, is found to be 1. 001 159 652 181 [9] to be compared with an experimental value equal to 1.001 159 652 180 [10]. The origin of the divergence of such series in QED is attributed to the punctual character of "lines" in the Feynman diagrams and have at least a possible solution in the framework of superstring theories when the "lines" of the Feynman diagrams are replaced by "tubes". This means that infinities in quantum electrodynamics might have and certainly have a deep physical meaning.

The question is then open to know whether the divergence of the Davis scheme is purely "accidental" or whether there is a deep physical meaning as well behind such a behavior.

4 References

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