

LUNEBERG'S INTEGRAL FOR THE DESCRIPTION OF INTERFEROMETRIC PARTICLE IMAGING BEYOND FRESNEL'S APPROXIMATION

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Abstract

Interferometric Particle Imaging is described using a Luneberg's formalism. It can predict and explain the results recorded when Fresnel conditions are not respected.

1 Introduction

ILIDS, and more generally Interferometric Particle Imaging, is an efficient technique to measure the size of droplets or irregular rough particles in a flow when the density of particles remains limited [1-3]. Assimilating particles to an ensemble of coherent emitters (two for a droplet in a classical ILIDS configuration, much more for rough particles), the image formation process can be accurately described using a Fresnel integral [4,5]. Unfortunately, it can happen that Fresnel conditions are not respected (for example with a large aperture combined to a relatively low defocus parameter), inducing some deformations of the interferometric images. This effect is easily identified in the case of droplets: the interference patterns are no more constituted of perfectly parallel fringes on the borders of the images. But it is more difficult to detect the problem in the case of rough particles whose interferometric images are speckle patterns. Without care, the analysis of the pattern could then lead to an erroneous reconstruction of the particle (in size and shape).

In order to understand the modifications that occur, a theoretical description of interferometric Particle Imaging that goes beyond Fresnel conditions is necessary. We propose a description based on the more general Luneberg integral [6]. It is first validated in the case of droplets in an ILIDS configuration, and then applied to rough particles.

2 Results

2.1 Spherical droplets observed with the ALIDS probe

Let us first consider the case of water droplets. The ALIDS probe is a prototype that has been realized within

Seventh framework program of the European Community to perform airborne measurements of the size of droplets (EUFAR project: European Facility for Airborne Research). It is been presented in reference [7]. The optical set-up of this probe consists of a frequency-doubled Quantel Ultra 100 laser emitting at 532 nm, a f/0.95 Goyo objective (focus length 25mm) associated to optical windows and deflecting mirrors that ensure a better compactness to the device, and a 8-bit DALSA Genie HM1024 CCD camera. Figure 1(a) shows the interferometric image of a droplet recorded in the highest aperture's configuration. The interference fringes are not perfectly vertical and parallel. We observe some curvature of the fringes, especially on the borders of the image. It is not possible to describe the geometry of the fringes using a classical simulator based on a Huygens-Fresnel integral [4]. But figure 1(b) shows the interferometric image of a 180 μm large droplet using the Luneberg's formalism that has been developed, according to the exact set of parameters of the ALIDS geometry. It appears that it is possible to predict the modifications observed experimentally, which would not be possible with a model based on Fresnel conditions which gives vertical fringes all over the image.

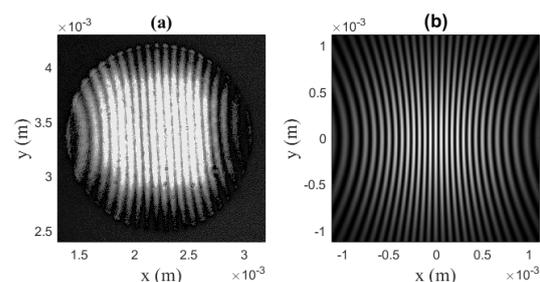


Figure 1 interferometric image of a droplet recorded with the ALIDS probe under highest aperture (a) and corresponding simulation using Luneberg's formalism and the set of parameters of the probe's (b)

2.2 Irregular rough particles

The interferometric image of irregular rough particles is more complex: it is indeed a speckle pattern [5,8-10]. Assuming that the scattering particle can be assimilated to an ensemble of coherent point emitters located on the envelope of the particle, it is possible to simulate the interferometric image of the particle performing generalized Huygens-Fresnel integrals [5]. In these Fresnel conditions, results show further that the 2D-Fourier transform of the interferometric image gives the 2D-autocorrelation of the envelope of the particle, which could be confirmed in various experiments. Both functions are linked by the scaling factor λB_{tot} , where λ is the wavelength of the illuminating laser and B_{tot} is the B-parameter of the optical transfer matrix of the imaging set-up (from the particle to the CCD sensor) [8]. In next simulations, $\lambda=532\text{nm}$ and $B_{\text{tot}}=2\text{mm}$. One can wonder what modifications will be observed if Fresnel conditions are not respected, as in previous section with droplets.

To illustrate this, figure 2(a) shows the repartition of 200 coherent point emitters randomly located on a dendrite-like particle. Axes are in meters: the size of the length of the particle is thus around $40\ \mu\text{m}$. Figure 2(b) shows its interferometric image predicted using a Fresnel formalism [5], while figure 2(c) shows its image using a Luneberg-based description. All other parameters are identical in both simulations. As for droplets, significant differences are observed on the borders of the two images.

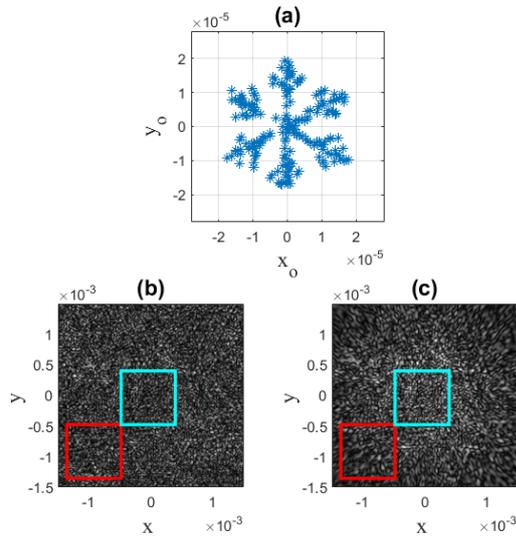


Figure 2 : dendrite-like particle composed of 200 coherent point emitters (a), its interferometric image using Fresnel model (b), and its interferometric image using Luneberg model (c). Axes are in meters.

In a second step, figure 3(a) shows the binarized 2D-Fourier transform of the blue box section of the pattern of

Fig 2(b) (Fresnel kernel), while figure 3(b) shows the binarized 2D-Fourier transform of the blue box section of the pattern of Fig 2(c) (Luneberg kernel). Both results are very similar. There is no difference between Fresnel and Luneberg models in the centers of the interferograms. However, for comparison, figure 4(a) shows the binarized Fourier transform of the red box section of the pattern of Fig 2(b) (Fresnel kernel), while figure 4(b) shows the binarized Fourier transform of the red box section of the pattern of Fig 2(c) (Luneberg kernel). Both are now very different. If the 2D-Fourier transform of this off-axis section of the interferometric pattern (red box) can still be assimilated to the 2D-autocorrelation of the particle using Fresnel model, Luneberg-based calculations show that this property is not respected if Fresnel conditions are not respected. Using this Fresnel-deduced result, we would deduce erroneously a smaller size and a different shape of the particle, and not the real ones.

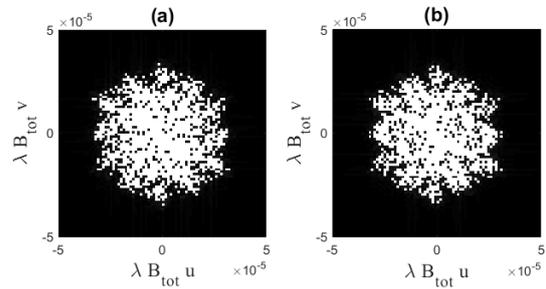


Figure 3 : binarized 2D-Fourier transform of the blue box section of pattern of Fig 2(b) (Fresnel kernel) (a), and binarized 2D-Fourier transform of the blue box section of pattern of Fig 2(c) (Luneberg kernel) (b). Axes are in meters.

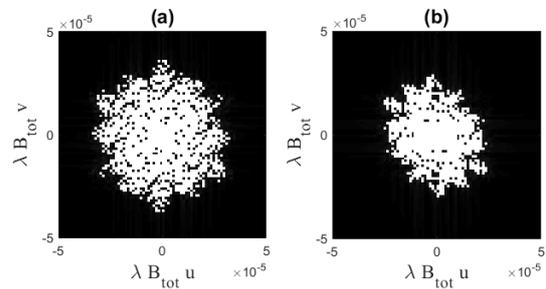


Figure 4 : binarized 2D-Fourier transform of the red box section of pattern of Fig 2(b) (Fresnel kernel) (a), and binarized 2D-Fourier transform of the red box section of pattern of Fig 2(c) (Luneberg kernel) (b). Axes are in meters.

In this work, a description of interferometric particle imaging (IPI) based on Luneberg conditions will thus be done and compared to experimental results. The modifications induced on rough particle imaging will be discussed for different shapes of particles. It will be

possible to discuss the 3D-tomography of particles [11], combining different angles of view, when Fresnel conditions are not respected. Finally, as this new formalism gives a vectorial description of the field, this potentiality of the formalism to better understand IPI experiments will be discussed.

3 References

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