

## RADIATION FORCE ON A PEMC SPHERE ILLUMINATED BY ARBITRARY-SHAPED BEAM

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### Abstract

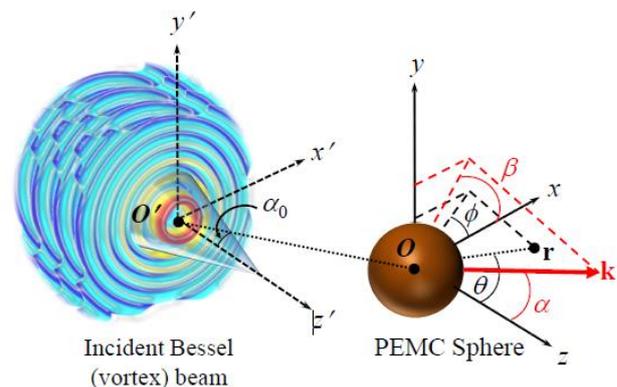
The radiation force generated by the interaction between the arbitrary-shaped optical polarized beams and the PEMC (perfect electromagnetic conductor) sphere is discussed in this paper. The generalized Lorenz-Mie theory (GLMT) for a laser beam of arbitrary shape illuminating a perfect electromagnetic conductor (PEMC) sphere is presented. The precise expressions of arbitrary-shaped optical polarized beams are attained based on the vector spherical wave functions (VSWFs) and beam shape coefficients (BSCs). The scattering beams are composed of the VSWFs and scattering coefficients which are gotten considering the special boundary conditions at the surface of the PEMC sphere. According to the Maxwell stress tensor, the exact expression of the radiation force in the GLMT frame is attained, which is divided into three parts: co-polarization, cross-polarization, and interference components. Taking the Bessel beam as incident, the effects of size parameters, admittance coefficient, and the beam parameters of the Bessel beam on the radiation force are calculated. The radiation force effect between the structural beam and the meta-material has promising application prospects, such as in biology, medicine, and particle manipulation.

### 1 Introduction

As a type of electric-magneto metamaterial [1, 2], the perfect electromagnetic conductor (PEMC) [3, 4] aroused more interest for its special characteristics. Different from the dielectric conductors, the electromagnetic field disappears in the internal of PEMC [5]. Besides, the "optical rotation" effect [6, 7] is presented when a PEMC is illuminated by some kind of polarized beam. Just as its name implies, the polarization states change when the electromagnetic field illuminates to a PEMC, which induces other cross-polarized scattering waves except for conventional co-polarized scattering waves. This "optical rotation" (It is also named circular dichroism.) effect has been discovered in chiral material [8, 9] (quartz and tartrate), liquid crystals [10, 11], and ice crystal [12] .etc. Besides, it has been applied to the pharmaceutical and biological fields [13].

The PEMC usually is considered as an extension of the perfect electrical conductor (PEC) [14] and perfect magnetic conductor (PMC) [15]. The characteristics both of

PEC and PMC are united by the PEMC using admittance coefficient  $M$ . The PEMC degenerates into PEC (limit  $M \rightarrow \infty$ ) or PMC ( $M = 0$ ) only depending on the  $M$ . Since the concept of PEMC was put forward, there are plenty of researches on the PEMC because of its promising prospect.



*Figure 1 Demonstration diagram that a PEMC sphere is illuminated by an arbitrarily shaped beam (The Bessel beam with half-cone angle  $\alpha_0$  as the incident beam in this Fig). The radial vector  $\mathbf{r}$  and the wave vector  $\mathbf{k}$  are shown in  $O - xyz$  coordinate. The included angle  $\alpha$  and  $\beta$  express the relation between the wave vector  $\mathbf{k}$  and the  $O - xyz$  coordinate.*

The PEMC is seen as a perfect reflector to the beam and the electromagnetic field. Therefore, the reflection, scattering, and absorption of PEMC to electromagnetic waves always is a key research area [16]. Over the past few years, the interaction when a plane - wave illuminates to the PEMC is a critical research direction. A research studied the scattering of two-dimensional PEMC and PEMC strips to the plane - wave [17]. Another research also studied the scattering that a plane - wave illuminates to a PEMC strip which buried in non-integer dimensional dielectric half-space is using Kobayashi potential method [18]. Some scholars made some researches that the electromagnetic radiation scattering by PEMC circular cylinder [19], sphere [20] and arbitrarily oriented dipole field [21] in free space. Another scholar studied the electromagnetic scattering by a PEMC plate in a lossy medium [22]. Besides, some studies considered the scattering effect [23] and polarizabilities when the electromagnetic wave illuminates a PEMC sphere located in chiral media. The scattering characteristics that the plane - wave illuminates to a PEMC cylinder covered by a

homogeneous plasma anisotropic material also has been studied [24]. What's more, the interference and diffraction effects also attract others' attention [25]. In recent years, a study focused on the diffraction when a plane - wave illuminates to a PEMC half-screen [26].

Recently, the research contents also extent to the radiation force and the torque effects, not just the scattering effect. The electromagnetic radiation force was studied when a plane - wave illuminates to a PEMC circular cylinder[28,29]. Later, the more complex model to research the electromagnetic radiation force was built that PEMC sphere is illuminated by a linear polarization plane - wave [30]. Besides, the radiation force and torque were explored when a circular polarized plane - wave illuminates to a PEMC sphere placed in a lossy space [31]. More physical characteristics have been discovered based on these relatively new researches.

Improvement based on the PEMC theory and combined the well-known characteristics of arbitrary-shaped optical polarized beams, this manuscript highlights the interaction between the PEMC sphere and the arbitrary-shaped optical polarized beams. The main formulas that arbitrary-shaped optical polarized beams illuminate to a PEMC are derived. At first, the beam shape coefficients (BSCs) of arbitrary-shaped beams are derived using the angular spectrum decomposition method (ASDM). By substituting the expressions of the vector spherical wave functions (VSWF), the exact expressions of arbitrary-shaped optical beams under different polarization states are attained. Later, the precise formulas for scattering fields are attained based on the generalized Lorenz-Mie theory (GLMT). Based on the correlation expressions of the incident and the scattering fields, both the longitudinal and the transverse radiation forces are determined. The scattering coefficients are gotten after solving the boundary conditions. Different from traditional scattering coefficients gotten when electromagnetic wave illuminates to a dielectric sphere, the scattering coefficients are divided into co-polarized and cross-polarized components. In order to better explain the physical phenomenon, this paper uses the Bessel beam as the incident illuminating to a PEMC sphere. The optical radiation force and its various polarization components are highlighted in this paper.

## 2 Method

The incident electromagnetic field are expressed as Eqs. (1 - 2) according to the VSWFs and BSCs.

$$\mathbf{E}_{inc} = -\sum_{n=1}^{\infty} \sum_{m=-n}^n iE_{mn} [p_{mn} \mathbf{N}_{mn}^{(1)}(k, \mathbf{r}) + q_{mn} \mathbf{M}_{mn}^{(1)}(k, \mathbf{r})] \quad (1)$$

$$\mathbf{H}_{inc} = -\frac{k}{\omega\mu_0} \sum_{n=1}^{\infty} \sum_{m=-n}^n E_{mn} [q_{mn} \mathbf{N}_{mn}^{(1)}(k, \mathbf{r}) + p_{mn} \mathbf{M}_{mn}^{(1)}(k, \mathbf{r})] \quad (2)$$

Where, the  $\mathbf{N}_{mn}^{(1)}$  and  $\mathbf{M}_{mn}^{(1)}$  are the VSWFs and the BSCs ( $p_{mn}$  and  $q_{mn}$ ) are attained based the ASDM. the scattering

electric field and magnetic field are expressed using VSWFs as

$$\mathbf{E}_{sca} = \sum_{n=1}^{\infty} \sum_{m=-n}^n iE_{mn} [a_{mn} \mathbf{N}_{mn}^{(3)}(k\mathbf{r}) + b_{mn} \mathbf{M}_{mn}^{(3)}(k\mathbf{r})] \quad (3)$$

$$\mathbf{H}_{sca} = -\frac{k}{\omega\mu_0} \sum_{n=1}^{\infty} \sum_{m=-n}^n E_{mn} [q_{mn} \mathbf{N}_{mn}^{(1)}(k, \mathbf{r}) + p_{mn} \mathbf{M}_{mn}^{(1)}(k, \mathbf{r})] \quad (4)$$

Where the  $a_{mn}$  and  $b_{mn}$  are the scattering coefficient. with

$$E_{mn} = i^n E_0 \sqrt{\frac{2n+1}{n(n+1)} \frac{(n-m)!}{(n+m)!}} \quad (5)$$

Because of the boundary condition of PEMC sphere surface:

$$\mathbf{n} \times (\mathbf{H} + M\mathbf{E}) = \mathbf{0} \quad (6)$$

The scattering coefficients can be attained:

$$\begin{aligned} a_{mn} &= a_{mn}^{TE \rightarrow TM} + a_{mn}^{TM \rightarrow TM} \\ b_{mn} &= b_{mn}^{TE \rightarrow TE} + b_{mn}^{TM \rightarrow TE} \\ a_{mn}^{TM \rightarrow TM} &= p_{mn} a_n^{TM \rightarrow TM} \\ a_{mn}^{TE \rightarrow TM} &= q_{mn} a_n^{TE \rightarrow TM} \\ b_{mn}^{TE \rightarrow TE} &= q_{mn} b_n^{TE \rightarrow TE} \\ b_{mn}^{TM \rightarrow TE} &= p_{mn} b_n^{TM \rightarrow TE} \\ a_n^{TM \rightarrow TM} &= \frac{\left\{ M^2 \eta^2 h_n^{(1)}(ka) [kaj_n(ka)] + j_n(ka) [kah_n^{(1)}(ka)] \right\}}{(1 + M^2 \eta^2) h_n^{(1)}(ka) [kah_n^{(1)}(ka)]} \\ a_n^{TE \rightarrow TM} &= \frac{iM\eta \left\{ j_n(ka) [kah_n^{(1)}(ka)] - h_n^{(1)}(ka) [kaj_n(ka)] \right\}}{(1 + M^2 \eta^2) h_n^{(1)}(ka) [kah_n^{(1)}(ka)]} \\ b_n^{TE \rightarrow TE} &= \frac{\left\{ M^2 \eta^2 j_n(ka) [kah_n^{(1)}(ka)] + h_n^{(1)}(ka) [kaj_n(ka)] \right\}}{(1 + M^2 \eta^2) h_n^{(1)}(ka) [kah_n^{(1)}(ka)]} \\ b_n^{TM \rightarrow TE} &= -a_n^{TE \rightarrow TM} \end{aligned} \quad (7)$$

Where the  $a_{mn}^{TM \rightarrow TM}$  and  $b_{mn}^{TE \rightarrow TE}$  are the scattering co-polarization components, and the  $a_{mn}^{TE \rightarrow TM}$  and  $b_{mn}^{TM \rightarrow TE}$  are the scattering cross-polarization components.

Because the time - averaged Maxwell's stress tensor is

$$\langle \mathbf{\sigma} \rangle = \left( \varepsilon_0 \mathbf{E} \otimes \mathbf{E} + \mathbf{H} \otimes \mathbf{H} - \frac{1}{2} (\varepsilon_0 \mathbf{E} \cdot \mathbf{E} + \mathbf{H} \cdot \mathbf{H}) \mathbf{I} \right) \quad (9)$$

The expression of radiation force is shown as follows

$$\langle \mathbf{F} \rangle = \iint_{\Omega} \langle \mathbf{\sigma} \rangle \cdot \mathbf{n} d\Omega \quad (10)$$

Therefore, the precise expressions of radiation force in x, y, and z directions and their components can be rewritten as the combinations of BSCs and scattering coefficients. Due to space limitation, the derivation will not be carried out here.

## 3 Discussion and conclusion

In this work, the radiation force generated because of the interaction between the PEMC sphere and arbitrary-shaped optical polarized beams is discussed. The size parameter  $ka$  and admittance coefficient  $M$  have been proved they take effect on the radiation force and its co-polarization, cross-polarization and interference

components. Besides, the effect from order  $l$  and half cone angle  $\alpha_0$  of the Bessel beam has been verified. Limited to space, we can't discuss it here. This work is of great significance in the research of optical tweezers, particle manipulation, and metamaterials.

#### 4 Acknowledgement

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