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Classical and Stimulated Doppler Effects and the Recoil Effect on a Moving Media Interface

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Abstract

Light refraction on moving surface of an increasing gas bubble is examined in the case of laser-trapping in an absorbing liquid solution. The treatment involves Doppler effect on moving walls of the bubble. At different stages of the bubble growth both classical (CDE) and stimulated Doppler effects (SDE) take place. It may be treated the former, CDE, as a recoilless limit of the energy-momentum conservation laws while the latter, SDE, is considered to be manifestation of the disbalance in the quantities of up and down stimulated transitions in some resonance two-level surface quantum system. Such disbalance is caused by the spontaneous transitions in this system. The recoil effect on a moving media surface is possible also as a result of a mass transfer process through the interface; its probability is bound to increase with the velocity of the surface movement.

1 Introduction

1.1 Light Pressure and Recoil Effect

Light or radiation pressure phenomenon is caused by the feature of momentum of running electromagnetic wave (in electrodynamics) or photon (in quantum mechanics). It is studied theoretically and experimentally over a century, but some questions are to be answered yet, the choice between Minkowski and Abraham definitions for photon momentum in a medium being among them [1]. In the case of normal incidence from vacuum on an immobile surface of an absorbing medium, the pressure of the light flux creates momentum $\hbar\cdot\omega_0\cdot(1+R)/c$ on average per photon, and the momentum direction coincides with the light beam wave vector, here $\hbar\cdot\omega_0$ being the energy of the incident photon. The same applies to a specularly reflecting interface at reflection coefficient $R=1$ and modification Δp_r of photon momentum is derived in the process:

$$\Delta p_r = \frac{2\cdot\hbar\cdot\omega_0}{c}\,. \tag{1}$$

But single-photon momentum modification Δp_t and its sign is not determined uniquely for a photon transmitted through the free-space surface of transparent dielectric [2]. Let a plane monochromatic wave falls on a coated immobile smooth interface of the transparent semi-infinite medium. As far as law of refraction takes place, a tangential component of the light particle momentum is preserved in its passage through the interface while the normal component is increased (under natural assumption of medium refractive index being $n>1$). Then, according to the momentum conservation law, surface and medium together have to be forced outwards. It is a recoil effect of a sort because of its opposite trend to the case of radiation pressure on a specular or absorbing surface. Evidently, most of manifestations of so-called "negative optical forces" [3, 4] are connected with this recoil effect. However, in the limit of a large medium mass $M\to\infty$ the refraction process gets recoilless. It is obvious, our treatment is consistent only with Minkowski definition of photon momentum, i. e. $p_t = n\cdot\hbar\cdot\omega_0/c$. Therefore it results in transmitted photon momentum modification Δp_t at normal incidence on the free-space surface of the transparent dielectric:

$$\Delta p_t = \frac{\hbar\cdot\omega_0}{c}\cdot(n-1)\,. \tag{2}$$

Let us note that changing $n\to -1$ [5] in Eq. (2) gives the value of Δp_r from Eq. (1) and minus sign corresponds to different directions of $\mathbf{\Delta p}_r$ and $\mathbf{\Delta p}_t$.

Formally, the ratio of Eqs. (1) and (2) is

$$\frac{\Delta p_r}{\Delta p_t} = \frac{2}{n-1}, \tag{3}$$

and it is the value that is important for our following presentation.

1.2 Classical Doppler Effect on a Moving Interface: Wave Approach

Let us briefly consider the question concerning the Doppler effect in refraction-reflection processes on a moving surface of a transparent medium. At normal incidence of electromagnetic wave, electric and magnetic field components of a composite incident-reflected-refracted field have to be continuous on the moving interface [6]. If v is a velocity of the interface movement in the normal direction ($\mathrm{v} << c$), then the Doppler frequency shifts (DFSh) $\delta\omega_r$ or $\delta\omega_t$ take place for the reflected or transmitted-refracted wave accordingly. In doing so:

$$\delta\omega_t = \omega_t - \omega_0 = \pm\frac{\mathrm{v}}{c}\cdot(n-1)\cdot\omega_0 \tag{4}$$

where plus corresponds to the increase of the frequency when the interface moves in the same direction as light does while movement in the opposite direction results in reduction of the frequency of the transmitted wave.

As earlier, changing $n \to -1$ in Eq. (4) results in the DFSh value $\delta\omega_r = \omega_r - \omega_0$ of the reflected wave. It is easy to define the ratio of DFSh values $\delta\omega_r$ and $\delta\omega_t$:

$$\left|\frac{\delta\omega_r}{\delta\omega_t}\right| = \frac{2}{n-1}, \qquad (5)$$

the latter coinciding with the value from Eq. (3).

To conclude the introduction, let us note that the experimental measurement of the DFSh ratio gives not only the refractive index n of the moving dielectric medium but also the recoilless ratio of the photon impacts (in the limit of the medium mass $M \to \infty$) during its reflection-refraction processes on the free-space interface of the transparent dielectric. Deviations of the DFSh ratio from the proper value from Eq. (5) may indicate the deviation of nature of the reflection-refraction processes from the classical one.

2 Classical and Stimulated Doppler Effects and the Recoil effect on a Moving Interface: Quantum Approach

It is possible to consider the Doppler effect on a moving interface from the quantum point of view. The photon-based refraction process is illustrated schematically in Figure 1.

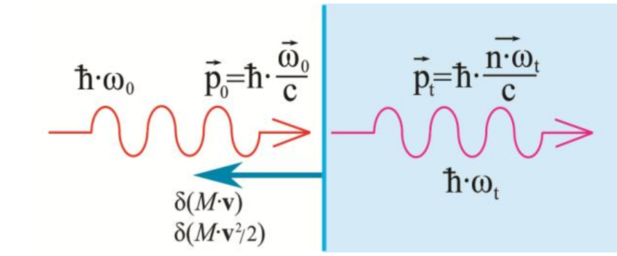

Figure 1 *The principal scheme of the photon refraction on a moving interface. Momentum-energy conservation laws must be fulfilled. A collinear geometry is not obligatory.*

If $\mathbf{v}$ is a velocity vector of the moving medium of the mass M then $\delta(M\cdot\mathbf{v})$ and $\delta(M\cdot\mathbf{v}^2/2)$ are correspondingly the recoil impact and energy originated at the photon transition through the interface. The following momentum-energy conservation laws must be fulfilled:

$$\mathbf{p}_0 = \mathbf{p}_t + \delta(M\cdot\mathbf{v})\,; \qquad (6)$$

$$\hbar\cdot\omega_0 = \hbar\cdot\omega_t + \delta(M\cdot\mathbf{v}^2/2) \qquad (7)$$

where $\mathbf{p}_0$ and $\mathbf{p}_t$ are momentums of the incident and refracted photon, $\hbar\cdot\omega_0$ and $\hbar\cdot\omega_t$ are corresponding energies. It is naturally to assume:

$$\delta(M\cdot\mathbf{v}) = M\cdot\delta\mathbf{v} + \mathbf{v}\cdot\delta M\,; \qquad (8)$$

$$\delta(M\cdot\mathbf{v}^2/2) = M\cdot\mathbf{v}\cdot\delta\mathbf{v} + \frac{\mathbf{v}^2}{2}\cdot\delta M\,. \qquad (9)$$

As photons are indivisible light particles, the plus signs perform rather disjunctive function in Eqs. (8) and (9). At $M = \text{const}$ only first summands are kept there and it results from Eqs. (6) and (7) that

$$\delta\omega_t = \omega_t - \omega_0 = \mathbf{v}\cdot\delta\mathbf{k}_t \qquad (10)$$

where $\delta\mathbf{k}_t = \mathbf{k}_t - \mathbf{k}_0$ is the wave vector alteration at the photon transition through the interface. In the recoilless limit $M \to \infty$ Eq. (8) provides $\delta\mathbf{v} \to 0$ and Eq. (10) describes CDE at refraction on a moving transparent interface. In the case of normal incidence it is easy to derive Eq. (4) from Eq. (10).

In experiments on laser thermocapillary gas bubble trapping some resonant absorption is inherent in the liquid medium [7]. Then the principal scheme in Figure 1 should be modified to the scheme in Figure 2. Separating two

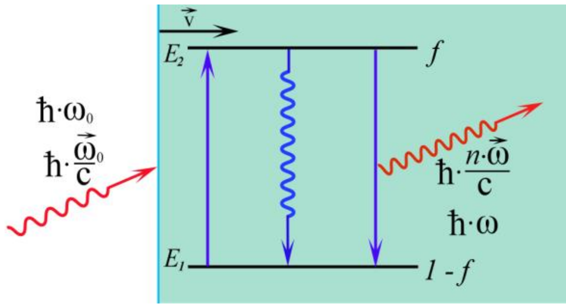

Figure 2 *The principal scheme of the photon refraction on a moving interface with resonant absorption. Straight blue arrows correspond to stimulated transitions, while waving blue – to spontaneous one:* ***f*** *and* ***1-f*** *are the level populations (in fractions of a unit) of upper and lower levels*

infinite media moving interface is treated as a quantum two-level surface system. Spontaneous and stimulated radiative processes are possible here. If $\delta\mathbf{k}_t^{st} = \hbar^{-1}\cdot\delta\mathbf{p}_t^{st}$ is the wave vector alteration at the stimulated photon transition through the interface then, as it easy to show by analogy with [8], the DFSh takes the "stimulated" value

$$\delta\omega_t^{st} = \hbar^{-1}\cdot\mathbf{v}\cdot\delta\mathbf{p}_t^{st}\cdot(1-f) = \mathbf{v}\cdot\delta\mathbf{k}_t^{st}\cdot(1-f) \qquad (11)$$

where $1-f$ has assumed the sense of the fraction of time the system spends in the lower state [8]. It is the name "stimulated Doppler effect" (SDE) that is proposed to describe the DFSh at the presence of stimulated emission and in accordance with Eq. (11).

Let us return to Eqs. (6)-(9). At $\mathbf{v} = \text{const}$ only second summands are kept in Eqs. (8) and (9). Having substituted them into Eqs. (7) and (6) we get

$$-\frac{\mathbf{v}^2}{2}\cdot\delta M = \hbar\cdot\delta\omega_t\,; \qquad (12)$$

$$\delta\omega_t = \omega_t - \omega_0 = \frac{1}{2}\cdot\mathbf{v}\cdot\delta\mathbf{k}_t\,. \qquad (13)$$

In the process a "mass defect" δM becomes a host of the recoil impact and energy per photon. That is why the term "recoil effect" (RE) will be used to present the process outlined by Eqs. (12)-(13).

Quantitatively, the distinctions between the Doppler and the recoil effects, CDE, SDE and RE, are characterised completely by Eq. (10), Eq. (11) and Eqs. (12), (13). But in a qualitative sense an analogy with certain classical mechanical model seems to be appropriate. Let some traveller to be jumping from a railroad bridge down onto a moving railway platform of a large mass M . If friction on the platform is large enough then the traveller successfully becomes a stowaway. Such transformation is realized at the expense of the recoilless transmission of momentum $M \cdot \delta \mathbf{v}$ from the platform to the passenger ($\delta \mathbf{v} \to 0$ is an infinitesimal reduction of the platform velocity, but $\mathbf{v} \neq \mathrm{const}$). To some extent, the described model is analogous to CDE at photon refraction process on a moving interface. But if there is no friction on the platform then the only way for the traveller to become a passenger is dumping some cargo mass δM from the platform, "Bolivar cannot carry double". In this case the velocity $\mathbf{v}$ of the platform remains the same due to lack of interaction force between the platform and the passenger. Evidently, such process is analogous to RE.

Generally, all of Eqs. (6-10, 12-13) could be repeated concerning the photon reflection on a moving interface. In doing so the only required modification is the replacement of the lower index *"t"* (transmitted-refracted) by the lower index *"r"* (reflected) in Eqs. (6-7, 10, 12-13). The question remains opened concerning a possibility of SDE (Eq. (11)) in reflection.

3 Experimental Observations of Doppler Effects on Moving Surface of a Gas Bubble Thermocapillary Trapped by a Laser Beam

Our foregoing discussion results from the experiments on laser thermocapillary trapping of a gas bubble in an absorbing liquid [9]. The He-Ne laser of power about 10 mW was used to trap a gas bubble in an ethanol solution coloured by brilliant green dye. The simple principal experimental scheme of the trap was presented in [7, 9]. It is possible to manage the size of the trapped gas bubble until it is too large. Movement of reflected and transmitted interference patterns observed may be treated as a manifestation of the Doppler effect because of the moving bubble walls.

The DFSh may be determined at any observation angle including both near-zero one and specular one by measuring the time dependences of intensities both in transmitted and reflected light. Therefore DFSh ratio defined by Eq. (5) is measurable. The examples of ordinary intensity dependencies are presented in Figure 3 at initial, intermediate and final stages of the bubble growth.

Evidently, at the initial stage (Figure 2 (a)) the proper DFSh ratio value of approximately 6 is observed. The table value of the refractive index of ethanol is $n = 1.360$. CDE takes place at this stage and refraction process is recoilless.

At the final stage of the bubble growth (Figure 2 (c)) the value of DFSh ratio is approximately two times more than the proper value from Eq. (5) [7, 9, 10], although the ripples of the reflected signal are not distinguishable enough in the Figure 2(c). Probably, it indicates a deviation from the classical recoilless limit, and SDE (at the state of saturated absorption $f \to 1/2$, see Eq. (11)) occurs in the photon refraction process on the moving bubble walls. There is a distinctive amplification of the scope of the transmitted signal oscillations at this stage, and this indicates contribution of stimulated radiation to the signal. The input of RE (see Eq. (13)) is possible also at this stage, but more likely its probability is essentially lower than the same of SDE.

The intermediate stage of the bubble growth is characterized by the specific form of a "cowboy hats" in the time dependence of the transmitted signal. The intermediate character of the stage provides non-zero probabilities for both CDE and SDE in photon refraction. Progressing from the initial stage to the final one increases SDE probability while diminishing probability of CDE.

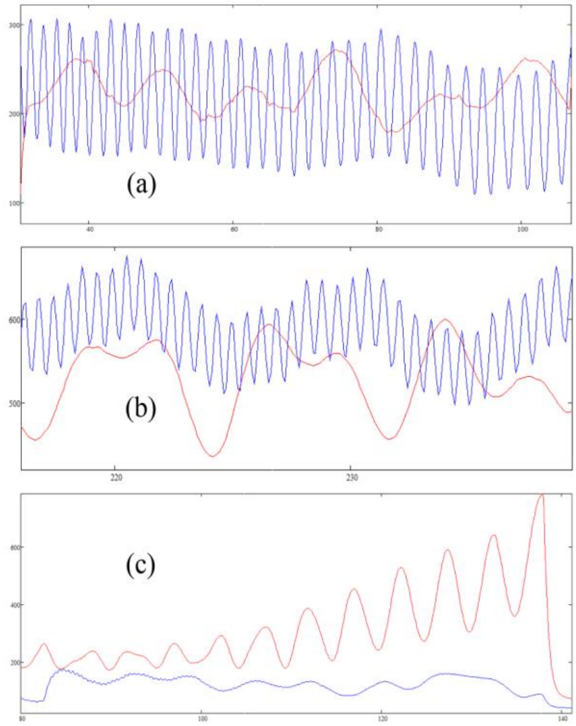

Figure 3 *The typical time dependences of the intensities (in arbitrary units) of the light transmitted through an increasing laser trapped gas bubble at near-zero observation angle (red curve) and of the light specularly reflected from it (blue curve): (a) at the initial trapping stage (initial bubble diameter* $D \approx 15 \div 20\ \mu m$ *); (b) at the intermediate stage (* $D \approx 50 \div 150\ \mu m$ *); (c) at the final stage (bubble escapes the trap at* $D \geq 200\ \mu m$ *). Time axis numbers are in seconds.*

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